

Fermion Loops, conserved currents and single-W

Alessandro Ballestrero ^a

^aI.N.F.N. and Dipartimento di Fisica Teorica, Torino

The relevance of fermion loop corrections to four fermion processes at e^+e^- colliders is reviewed with regard to the recent extension to the case of massive external particles and its application to single-W processes.

1. Introduction

The problem of preserving gauge invariance in the calculations involving unstable particles is well known since several years [1, 2, 3, 4] . It is connected with the fact that the use of a width (e.g. $i\Gamma M$) in the denominator of an s -channel unstable boson propagator is necessary to prevent divergences at $p^2 = M^2$, but it is in fact an effective way of including only a part of higher order corrections, and it therefore violates gauge invariance.

There are various examples in which this violation becomes numerically relevant. For instance, this happens at tree level with four fermion final states for WW production at high energies, where gauge cancellations among the three double resonant (CC03) diagrams become relevant, and for contributions at low e^- angle. This last case is relevant for final states like $e^-e^+ \rightarrow e^-\bar{\nu}_e u \bar{d}$, $e^-e^+ \rightarrow e^-\bar{\nu}_e \mu^+ \nu_\mu$ or $e^-e^+ \rightarrow e^-e^+ \nu_e \bar{\nu}_e$ with e^- undetected, which are commonly referred to as single-W processes. For them, gauge invariance determines the behaviour of the amplitude as a function of t .

In the above mentioned examples even small violations of gauge invariance may easily give results which are completely unreliable, and differ from the correct ones not by a few percent but by some large factor.

Various gauge restoring methods have been described in the literature and have been used to avoid such inconsistencies. Some of the most used are :

Fixed width(FW): In all massive-boson propagators one performs everywhere the substitu-

tion $M^2 \rightarrow M^2 - i\Gamma M$. This gives an unphysical width in t -channel, but retains U(1) gauge invariance.

Complex Mass(CM): The substitution $M^2 \rightarrow M^2 - i\Gamma M$ is applied not only in propagators but also in relations involving couplings. This gives unphysical complex couplings in addition to the width in t -channel. It has however the advantage of preserving both U(1) and SU(2) Ward identities.

Overall scheme(OA): When resonant propagators are present, all diagrams (not only resonant ones) are multiplied by $(q^2 - M^2)/(q^2 - M^2 + iM\Gamma)$. This method retains gauge invariance but mistreats non resonant terms.

Fermion Loop(FL): At least the imaginary part of all (propagators and vertices) fermion loop corrections is used together with resummed boson propagators. The real part of FL corrections can as well be considered. It is not necessary for gauge restoration, but it constitutes an important gauge invariant subset of the radiative corrections, which automatically determines the correct evolution of the coupling constants.

It has to be noticed that the first schemes are somehow “ad hoc” prescriptions to restore gauge invariance introducing some non correct feature, whose consequences are expected to be of little numerical impact but this should in principle be verified case by case. FL on the contrary is the only fully consistent and justified scheme in field theory.

In the following we will review the FL scheme and its application to single-W processes, with particular regard to the latest results which take into account current non conservation (massive external fermions).

2. Fermion Loop

The application of FL corrections to 4f processes has been fully described in the papers of ref. [1, 4]

In the first paper only the Imaginary part of FL corrections (IFL) have been considered and as a case study the process $e^-e^+ \rightarrow e^-\bar{\nu}_e u \bar{d}$ has been used. The approximation of considering massless external fermions has been taken, which implies that all external current are conserved (CC).

In this approximation U(1) Ward Identities are satisfied by adding a fixed width $i\Gamma M$ to all denominators of W propagators (also in t -channel), even if there is no physical justification for this procedure.

IFL corresponds to resumming the imaginary part of fermionic loop corrections in propagators and adding the imaginary part of vertex loop corrections. This has to be considered as the minimum set of corrections that has to be added in order to restore gauge invariance.

With massless fermions in the internal loops and in the decay width, $i\frac{\Gamma p^2}{M}$ in propagators satisfies gauge invariance.

Some numerical applications for $e^-e^+ \rightarrow e^-\bar{\nu}_e u \bar{d}$ have been considered in [1].

The complete FL corrections have instead been computed in ref [4]. Again in massless external fermion approximation, but with massive fermions in loops. The renormalization of gauge boson masses at their (gauge invariant) complex poles [5] has been used. It turns out that in this scheme all corrections can be reabsorbed in running couplings and renormalized propagator functions and triple vertices, so that an effective Born prescription can be used in which only tree level type diagrams appear.

Numerical applications to $e^-e^+ \rightarrow e^-\bar{\nu}_e u \bar{d}$ at low e^- angle and $e^-e^+ \rightarrow \mu^-\bar{\nu}_\mu u \bar{d}$ at high energies show differences from other schemes. At high energies differences also with IFL are found.

3. Single W

Processes with at least one electron and one electron-neutrino like $e^-e^+ \rightarrow e^-\bar{\nu}_e u \bar{d}$, $e^-e^+ \rightarrow e^-\bar{\nu}_e \mu^+ \nu_\mu$, and $e^-e^+ \rightarrow e^-e^+\bar{\nu}_e \nu_e$, besides being relevant to WW or ZZ physics, are particularly interesting in the kinematical configuration in which the electron is lost in the pipe. In such a configuration (single-W), they become important as a background to searches and for anomalous coupling studies. The cross sections are significant because of t -channel contributions, and they are directly measured at LEP2.

Single-W processes are divergent in massless external fermion approximation. Therefore not only external electron masses have to be exactly accounted for, but also those of the other fermions (u, d, μ, \dots).

Several fully massive 4fermion MonteCarlo's are now available: COMPHEP [6], GRC4F [7], WPHACT [8], KORALW [9], NEXTCALIBUR [10], SWAP [11] and WTO [12] which accounts for masses where they become important.

Good technical agreement among all these codes has been achieved for single-W processes in tuned comparisons. In fig. 1 one can find the results from the first three codes up to Linear Collider energies, while we refer to Ref. [13] for further comparisons with the others at LEP2 energies. It has to be remarked that in fig. 1 the three codes used respectively overall, $L_{\mu\nu}$ transform method [14] and fixed width as gauge restoring schemes.

4. IFL and Non Conserved Currents

FL calculations of ref. [1, 4] are not appropriate for single-W processes because of the assumption of conserved currents (massless external fermions), which is in conflict with the necessity to account for external fermion masses.

Nevertheless, numerical studies have been performed using these results together with fully massive matrix elements in [15], where it was noticed that the corresponding U(1) gauge violation is proportional to m_e . It can however be enhanced by large factors at high energy, as it will be shown in table 1 .

The Imaginary Fermion Loop scheme with fully

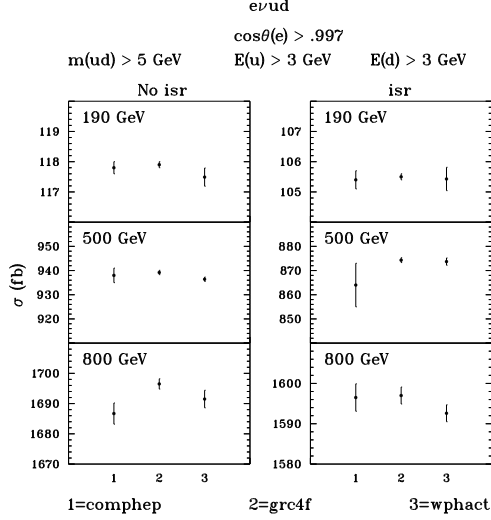


Figure 1. Comparison of the results from three codes for the single-W process $e^-e^+ \rightarrow e^-\bar{\nu}_e u\bar{d}$.

massive ME and exact non conserved current contributions has been studied recently [16] and implemented in WPHACT.

The unitary gauge has been used. While for massless internal fermions the Ward Identities (WI) are satisfied by the fixed width propagator

$$\frac{-i}{p^2 - M^2 + i\Gamma M_W} \left\{ g^{\mu\nu} - \frac{p^\mu p^\nu}{M^2 - i\Gamma M_W} \right\},$$

the correct resummed propagator is instead

$$\frac{-i}{p^2 - M^2 + i\Pi} \left\{ g^{\mu\nu} - \frac{p^\mu p^\nu}{M^2} \left(1 + \frac{i\Pi}{p^2} \right) \right\}$$

with “running width” $\Pi = p^2 \frac{\Gamma_W}{M_W}$

With it, WI are properly satisfied only if exact IFL triple vertex corrections are computed.

As it can be deduced from tables 1, 2, the use of IFL does not give significant numerical differences with “ad hoc” schemes for cross sections with typical single-W cuts. The last line of table 1 shows instead that the approximation of conserved currents together with massive ME lead to inconsistent numerical results at high energies.

Some differences between IFL and other schemes is evident (fig 2) when one considers mass distributions. This is connected to the fact that with IFL one properly makes use of the running width.

Table 1

Cross sections for $e^+e^- \rightarrow e^-\bar{\nu}_e u\bar{d}$ and various gauge restoring schemes, all implemented in WPHACT. No ISR. $M(u\bar{d}) > 5$ GeV, $E_u > 3$ GeV, $E_{\bar{d}} > 3$ GeV, $\cos(\theta_e) > .997$

	190 GeV	800 GeV	1500 GeV
IFL	.11815 (13)	1.6978 (15)	3.0414 (35)
FW	.11798 (11)	1.6948 (12)	3.0453 (41)
CM	.11791 (12)	1.6953 (16)	3.0529 (60)
OA	.11760 (10)	1.6953 (13)	3.0401 (23)
IFLCC	.11813 (12)	1.7987 (16)	5.0706 (44)

Table 2

Comparison of FW and IFL schemes for different single-W cross sections at $\sqrt{s} = 200$ GeV. No ISR. $|\cos\theta_e| > 0.997$, $E_\mu > 15$ GeV, and $|\cos\theta_\mu| < 0.95$. First line $M_{u\bar{d}} > 45$ GeV, second line $M_{u\bar{d}} < 45$ GeV

Final state	IFL	FW
$e^-\bar{\nu}_e u\bar{d}$	0.12043 (10)	0.12041 (11)
$e^-\bar{\nu}_e u\bar{d}$	0.028585 (14)	0.028564 (14)
$e^-\bar{\nu}_e \mu^+ \nu_\mu$	0.035926 (34)	0.035886 (32)
$e^-\bar{\nu}_e e^+ \nu_e$	0.050209 (38)	0.050145 (32)

5. FL and Non Conserved Currents

Big theoretical uncertainties for single-W processes, in absence of complete $O(\alpha)$ corrections, are connected to the scales of the couplings and to the scale for ISR with dominating t -channel contributions.

Complete FL calculations with massive external fermions have been recently performed[17]. These are necessary to solve the first of the uncertainties just mentioned.

Complex mass renormalization has been used as in [4] and it leads again to effective Born calculations. Besides running couplings and renormalized propagators now also running boson masses are needed. They are defined by

$$\frac{1}{M^2(p^2)} = \frac{1}{M^2} \frac{p^2 - S_W^0 + \frac{M^2}{p^2} S_\phi}{p^2 - S_\phi}$$

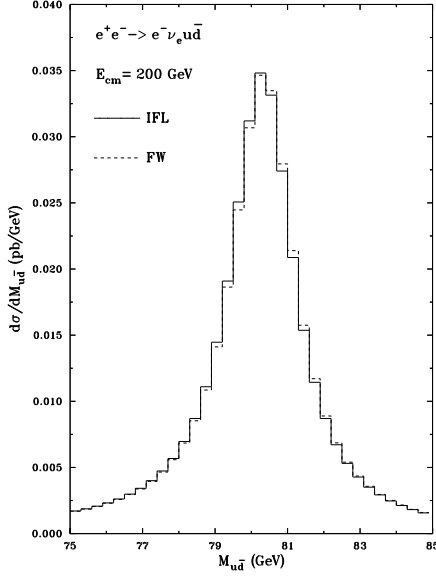


Figure 2. Mass distribution of the $u\bar{d}$ pair computed by WPHACT for fixed with and imaginary fermion loop schemes. No ISR. $|\cos\theta_e^-| > 0.997$

which for a massless internal world reduces to

$$M^2(p^2) = \frac{g^2(p^2)}{g^2(p_W)} p_W, \quad M^2(p_W) = p_W.$$

$$\frac{1}{g^2(s)} = \frac{1}{g^2} - \frac{1}{16\pi^2} \Pi_{3Q}(s)$$

It has to be remarked that, using the Feynman gauge, complete one loop resummation gives a W-propagator which is equivalent to some effective “unitary gauge” form:

$$\Delta_{\text{eff}}^{\mu\nu} = \frac{1}{p^2 - M^2 + S_W^0} \left[\delta^{\mu\nu} + \frac{p^\mu p^\nu}{M^2(p^2)} \right].$$

The complete FL scheme for non conserved currents has been implemented in WTO for $e^-e^+ \rightarrow e^-\bar{\nu}_e u\bar{d}$ and $e^-e^+ \rightarrow e^-\bar{\nu}_e \mu^+ \bar{\nu}_\mu$, where the fermion masses are accounted for “when needed”: in t -channel γ exchange diagrams for $\log(m^2/s)$ and constant contributions.

The numerical results [17] show differences with tree level fixed width G_f scheme up to $\sim 7\%$. This result is reported in fig 3.

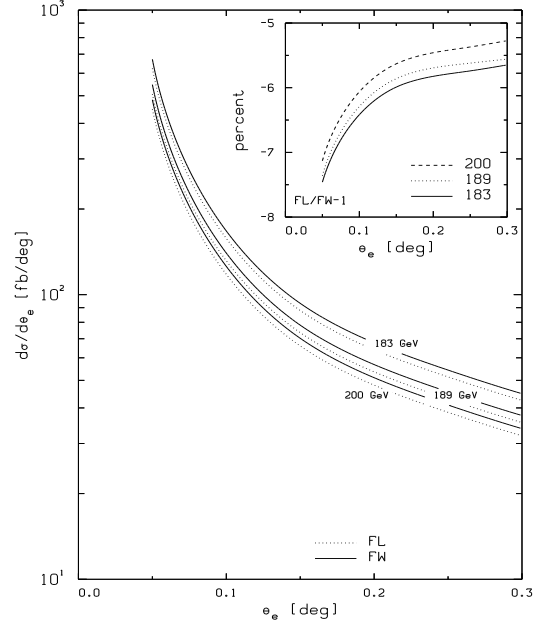


Figure 3. $d\sigma/d\theta_e$ distribution computed by WTO for $e^-e^+ \rightarrow e^-\bar{\nu}_e u\bar{d}$ and $M(u\bar{d}) > 45$ GeV

One may expect that the bulk of such a difference comes from α running effects, as it is rather obvious that the value of α_{em} in G_f renormalization scheme is not the correct one to describe t -channel γ propagator couplings. For such a reason, $\alpha(t)$ with IFL (IFL α) has been implemented in WPHACT for t -channel contribution only.

For cuts used at LEP2 energies for single-W, this seems to describe with very good approximation $e^-e^+ \rightarrow e^-\bar{\nu}_e u\bar{d}$, as can be deduced by a comparison with FL both for cross sections (table 3) and for angular distributions (table 4).

Table 3

Total single- W cross-section in fb for $e^+e^- \rightarrow e^-\bar{\nu}_e u\bar{d}$, for $M(u\bar{d}) > 45$ GeV and $|\cos\theta_e| > 0.997$. No ISR. FL is computed by WTO, IFL and IFL α by WPHACT.

\sqrt{s}	IFL	IFL α	FL
183 GeV	88.50(4)	83.26(5)	83.28(6)
189 GeV	99.26(4)	93.60(9)	93.79(7)
200 GeV	120.43(10)	113.24(8)	113.67(8)

Table 4

$d\sigma/d\theta_e$ [pb/degrees] for $e^+e^- \rightarrow e^-\bar{\nu}_e u\bar{d}$. $M(u\bar{d}) > 45$ GeV, $\sqrt{s}=200$ GeV. No ISR. FL is computed by WTO, IFL and IFL α by WPHACT.

θ_e [Deg]	IFL	IFL α	FL
$0.0^\circ \div 0.1^\circ$	0.67077	0.62404	0.62357
$0.1^\circ \div 0.2^\circ$	0.09321	0.08753	0.08798
$0.2^\circ \div 0.3^\circ$	0.05455	0.05141	0.05141
$0.3^\circ \div 0.4^\circ$	0.03867	0.03624	0.03646

Moreover, it has been checked in fig. 4 that the agreement does not depend on the $M(u\bar{d})$ invariant mass cut and it remains below 1% down to the very low cuts.

For the process $e^-e^+ \rightarrow e^-\bar{\nu}_e\mu^+\nu_\mu$ instead, the difference between FL and IFL α turns out to be of the order of 2-3% both in cross sections (table 5) and in angular distributions (table 6). The reason for such a different behaviour between the two processes is probably due to the cuts and the relative importance of multiperipheral contributions in them. The results of tables 5,6 in any case indicate that the running of α is not in general sufficient for a very accurate description of the effects accounted for by complete FL calculations

Table 5

Total single- W cross-section in fb for $e^+e^- \rightarrow e^-\bar{\nu}_e\mu^+\nu_\mu$, for $|\cos\theta_e| > 0.997$, $E_\mu > 15$ GeV, and $|\cos\theta_\mu| < 0.95$. No ISR. FL is computed by WTO, IFL and IFL α by WPHACT.

\sqrt{s}	IFL	IFL α	FL
183 GeV	26.45(1)	24.90(1)	25.53(4)
189 GeV	29.70(2)	27.98(2)	28.78(4)
200 GeV	35.93(4)	33.85(4)	34.97(6)

In fig. 5 one can see the differences of IFL and IFL α angular distributions for various processes, including $e^+e^- \rightarrow e^+e^-\nu\bar{\nu}$ where FL corrections are not available yet.

For the latter, the estimate by WPHACT of the theoretical uncertainty of IFL α calculations is of the

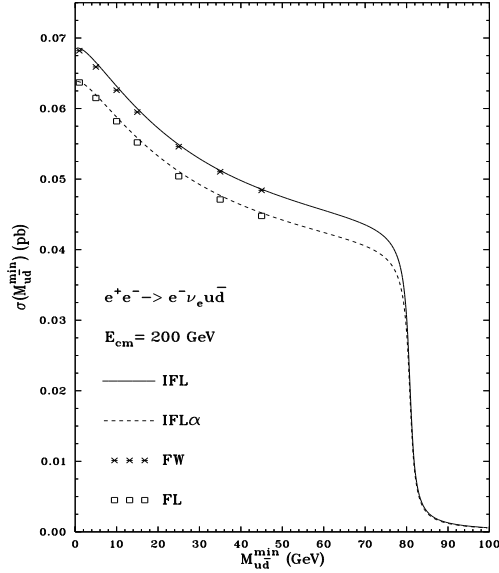


Figure 4. Total cross section for $\theta_e < 0.1^\circ$ as a function of the lower cut on $M_{u\bar{d}}$ in IFL and IFL α schemes. Curves computed with WPHACT. Markers give the results of FW and FL by WTO.

order of about 3% [13]. This refers only to the uncertainty connected to the absence of complete FL calculations and not to the one due to the treatment of ISR/FSR in presence of dominant t -channel contributions [13].

6. Conclusions

FL corrections have been extended to massive external fermions in 4f physics. With them calculations can be safely performed down to $\theta_e = 0$.

Single- W processes represent one of the most important applications of such extension.

IFL corrections show that “ad hoc” gauge restoring schemes are reliable for total cross sections but may produce some differences in distributions.

Complete FL are a gauge invariant subset of radiative corrections, essential for single- W processes.

Their results differ from G_f scheme by several percent.

IFL + α running for t -channel reproduces FL re-

Table 6

$d\sigma/d\theta_e$ in [pb/degrees] for $e^+e^- \rightarrow e^-\bar{\nu}_e\nu_\mu\mu^+$, for $|\cos\theta_e| > 0.997$, $E_\mu > 15$ GeV, and $|\cos\theta_\mu| < 0.95$. $\sqrt{s} = 183$ GeV. No ISR. FL is computed by WTO, IFL and IFL α by WPHACT.

θ_e [Deg]	IFL	IFL α	FL
$0.0^\circ \div 0.1^\circ$	0.14170	0.1319	0.13448
$0.1^\circ \div 0.2^\circ$	0.02117	0.01987	0.02031
$0.2^\circ \div 0.3^\circ$	0.01240	0.01166	0.01194
$0.3^\circ \div 0.4^\circ$	0.00879	0.00830	0.00851

sults at less than 1% level for $e^-e^+ \rightarrow e^-\bar{\nu}_e\mu^+\nu_\mu$ and at 2-3% for $e^-e^+ \rightarrow e^-\bar{\nu}_e\mu^+\nu_\mu$.

Further theoretical uncertainties for single-W, connected with ISR in t -channel dominated processes have not been considered in this short account.

Further analyses and improvements are probably still needed for single-W processes, especially at Linear Collider.

Acknowledgments

I wish to thank Johannes Blümlein and Tord Riemann for the pleasant and constructive atmosphere at the 5th Zeuthen Workshop “Loops and Legs in Quantum Field Theory”.

I am particularly grateful to Elena Accomando and Ezio Maina for their collaboration.

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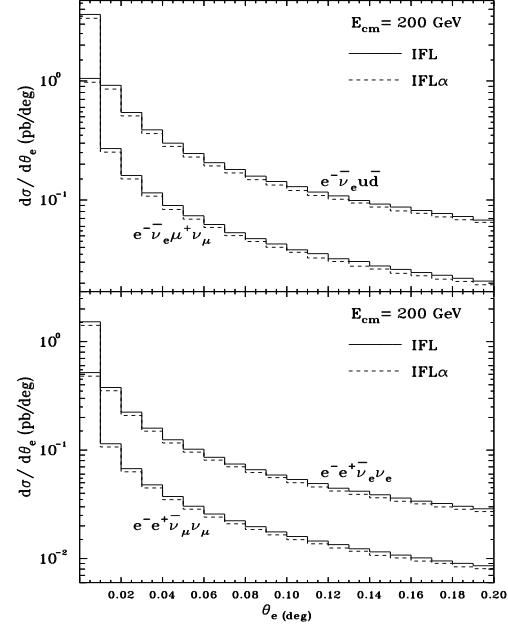


Figure 5. Angular distributions for different single-W processes at $\sqrt{s} = 200$ GeV in the IFL and IFL α scheme.

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